Drawing Method Combined With Matrix Representation and Anchored Map for Semi-Bipartite Graph

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1. Introduction

Bipartite graph is a kind of graph whose nodes can be divided into two disjoint sets A and B such that every edge connects a node in A to one in B. If there are another kind of edges exists within one node set, we call it *semi-bipartite graph* (Figure 1).Social networks are typical examples of semi-bipartite graphs: there are "friendships" between people and communities they belong to. In this case, A would be the set of communities and B the set of people. The definition of semi-bipartite graph is as follows:

$G = (A \cup B, E_1 \cup E_2), A \cap B = \phi$

$E_1 \subseteq \{(u,v) \mid u \in A, v \in B\}$

$E_2 \subseteq \{(u,v) \mid u, v \in B\}$ or $E_2 \subseteq \{\{u,v\} \mid u, v \in B\}$

(Where E_2 can be both direct and indirect)

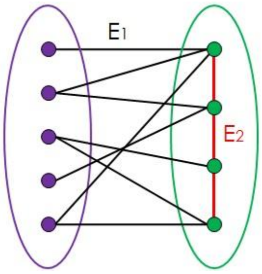

Figure 1.Semi-bipartite graph example

In this paper, a drawing method combined with matrix representation [1] and anchored map [2] for visualizing semi-bipartite graph is presented.

2. Related Work

2.1 Matrix Representation

Henry et al. [1] presented a hybrid representation that combines with two traditional representations: node-link diagram and adjacency matrix (Figure 2). The adjacency matrix of a graph G with n nodes is the $n \times n$ matrix where the non diagonal entry a_{ij} represent if there is an edge from node i to node j which makes matrix a useful tool to support both directed and undirected graph. Additionally it has been proved that the adjacency matrix representation is particularly effective for dense graphs.

2.2 Anchored Map

Anchored map [2] is a node-link diagram based on spring model which is used to visualize bipartite graphs. It has been proposed and proved as an effective drawing technique to acquire the knowledge from bipartite graphs.

Nodes in one set are called "anchors" which are arranged on a circumference, and nodes in another set are arranged at suitable positions in relation to adjacent anchors.

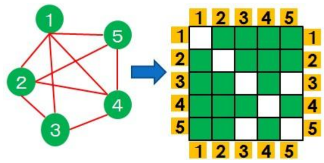

Figure 2 Left: Node-link diagram and matrix representation

3. Drawing Method

3.1 Approach of Our Drawing Method

Original algorithm of anchored map is not applicable to visualize semi-bipartite graph directly because E_2 edges are not considered in its algorithm. But since the only difference between these two graphs is that there are edges within node set B, we considered that anchored map style can be performed if E_2 could be handled.

We tried to transform semi-bipartite graph into bipartite graph which has not succeeded but still we found that after clustering the nodes of set B into some node clusters by the relation of E_2. The number of edges between node clusters can be greatly reduced. Then node clusters can be represented by matrix style and treated as free nodes of anchored map.

Our drawing method is to combine these two techniques together for visualizing semi-bipartite graph (Figure 4: right).

3.2 Aesthetic Criteria

We consider the following criteria for the visualization of semi-bipartite graph:

(R_1) Anchors which connected to common free nodes should be laid out as closely as possible.

(R_2) Free nodes which connected to each other should be laid out as closely as possible

(R_3) Reduce the sum of lengths of edges

(R_4) Reduce the number of edge-crossing.

4. Drawing Procedure

4.1 Node Clustering

As discussed before, at first we will apply on the node sets B the clustering algorithm provided by Newman [3] transforming nodes of B into some node clusters (E_2 edges within the groups of nodes are dense but sparse between them) which will be represented as matrices, while node cluster with one node will be treated as a single node.

4.2 Arranging the Node Set A

Anchored map proposed an algorithm for finding the optimal order of anchor nodes with minimum edge-crossing. For semi-bipartite graph, since E_2 edges are also existed and their crossing will be also influenced by the order of anchors. Therefore the algorithm of original anchored map does not work. To overcome this problem, we developed an algorithm based on the original one which is capable of finding the optimal order of anchor nodes while E_2 edges exist.

But our algorithm cannot work well while there are lots of E_2 edges. And that is exactly why we use node clustering to reduce the number of E_2 edges.

4.3 Arranging the Node Clusters

The same as anchored map, node clusters are arranged at suitable positions in relation to adjacent anchors based on spring embedded model. And after node clustering, the edge between one anchor and node cluster will be multiple. This is considered when arranging the node clusters in our research.

4.4 Representing Node Cluster by Matrices

The node clusters are visualized as shown in Figure 3, and we found that the order of nodes within the matrix will change the edge-crossing (Figure 3 left), so in our research how to found the order with minimum edge-crossing is considered.

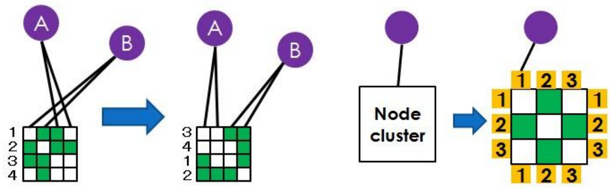

Figure3. Left: different order with different edge-crossing
Right: matrix representation

4.5 Drawing edges

We choose straight line to represent edge. There are 4 connecting points for each node within the matrix (Figure 3 right). Simply we choose the nearest point.

5. Example

We choose a data of social networks as an example to show the features of proposed drawing style. Sample data includes 8 communities (anchors), 43 people (free nodes), 46 E_1 edges (community - people) and 42 E_2 edges (friendship).

Figure 4 left shows the result of anchored map style (treating E_2 the same way as E_1).The right one is the result of our drawing method where the number of free nodes was reduced to 12 (8 clusters and 4 singles node) and only 7 E_2 edges left. The displaying part is not implemented, so relationship between nodes within matrices cannot be seen and edges between anchor and matrix are all single. But actually multiple edges exist as illustrated in Figure 3 left shows, and the position of all matrices are calculated from these multiple edges.

Even though matrices cannot be seen yet, the features of our research can still be understood by just imaging the squares are displayed as matrices (as Figure 2 and 3). So the relationship between nodes within matrices can be easily understood and by observing the edges, the relation outside matrices can also be understood. And since there are four connecting points for each node within matrix (Figure 3 right). The edge-crossing can be effectively reduced providing better readability.

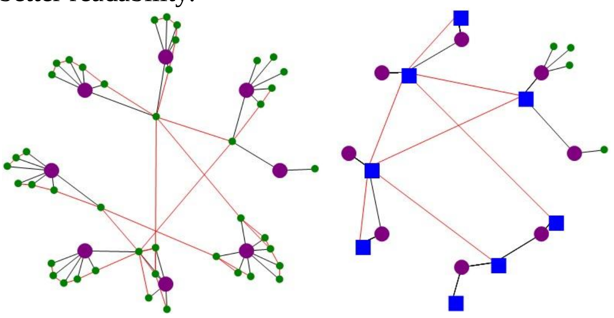

Figure 4.Left: Anchored map style. (E_1 E_2 treated as the same)
Right: proposed drawing method of our research

6. Conclusions

In this paper, we present a drawing method combined with matrix representation and anchored map to visualize semi-bipartite graph. Our drawing method can visualize the two kinds of relationship (edges) in one graph with good readability, and using our method, not only direct but also potential information can be seen. For example, people will be clustered as some groups by their relationship (algorithm by Newman [3]). By this way some key persons (connect two friend-groups together) can be found, at the same time, the reason why they become friends can be understood by looking at their common-community information (be in the same university and so on)

References

[1] N.Henry, J.D.Fekete, M.J.McGuffin. NodeTrix: A Hybrid Visualization of Social Networks. In Proceedings IEEE Symposium on Information Visualization, pp.1302–1309, 2007.
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[3] M. E. J. Newman. Fast algorithm for detecting community structure in networks. Physical Review E 69, 066133, 2004.